

Progression in Calculations

Addition

Year 1			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p> <p>Basic to subject specific (Beck's Tiers): +, add, more, plus, make, sum, total, altogether, score double, near double one more, two more... ten more how many more to make...? how many more is... than...? how much more is...?</p> <p>Instructional vocabulary: start from, start with, start at, look at, point to, show me</p>	<p>Use cubes to add two numbers together as a group or in a bar.</p>	<p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p> <p>Use the part-part whole diagram as shown above to move into the abstract.</p> <p>$4 + 7 + 6 = 10 + 7 = 17$</p>
<p>Counting objects, partitioning and recombining sets using practical apparatus.</p> <p>Understand that the number gets bigger.</p> <p>Pictorial recording of practical experiences.</p> <p>Modelling of commutative layout. ($3+6=9$, $6+3=9$)</p> <p>Counting on from the larger number.</p>			



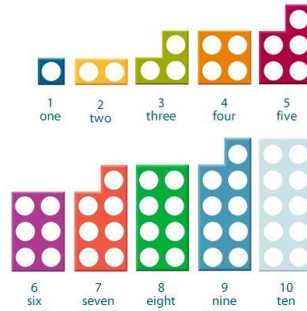
$1 + 1 = 2$
double 1 is 2

$2 - 1 = 1$
half of 2 is 1



$2 + 2 = 4$
double 2 is 4

$4 - 2 = 2$
half of 4 is 2

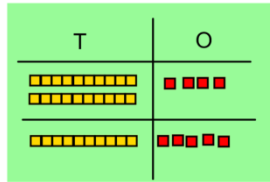


Recognition of quantities. Find the larger number first.

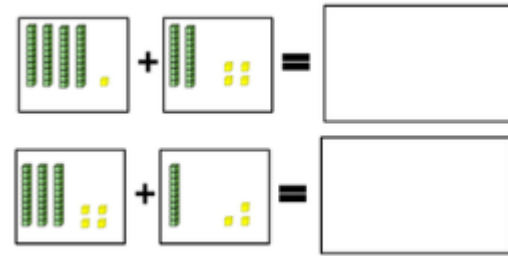
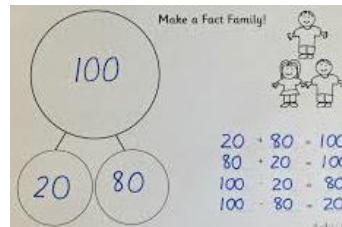
$5 + 8$ is re-ordered to $8 + 5$.
Count on from 8.
Therefore, $8 + 5 = 13$

Year 2			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p> <p>Key skills of knowing number bonds to 10 and within 20.</p> <p>Regrouping to make bonds to 10.</p> <p>Develop knowledge of fact families, e.g. 2, 5, 7.</p> <p>All answers to be recorded in a number sentence following any informal recording.</p> <p>Understand the effect of adding a zero.</p> <p>Add a 2 digit number and units.</p>	<p>Basic to subject specific (Beck's Tiers): +, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make...? how many more is... than...? how much more is...?</p> <p>Instructional vocabulary: tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you...</p> <p>This shows regrouping of 9 + 3 to become 10 + 2.</p> <p>This shows the fact family of 10, 7 and 3.</p>	<p>Use pictorial representations. Regroup or partition the smaller number to make 10.</p> <p>Fact families.</p> <p>9+1=10 1+9=10</p> <p>100 = 80 +20 100 = 20 + 80</p>	<p>7 + 4= 11</p> <p>If I am at seven, how many more do I need to make 10?</p> <p>7 + 8 = 15 15 = 8 + 7</p> <p>If I add zero to any number, the number stays the same.</p> <p>16 + 0 = 16</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>34 + 23 = 57</p> <p>30 + 20 = 50</p> <p>4 + 3 = 7</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>34 + 19 is the same as 33 + 20.</p> </div>

Add a 2 digit number and tens.
Add two 2 digit numbers.


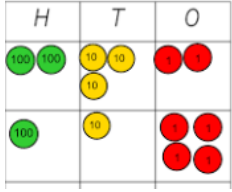


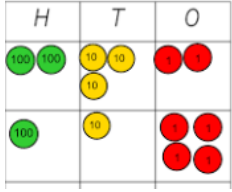



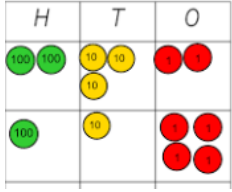




Build the 2 digit numbers using apparatus such as dienes.



Pupils to record their own 'sticks and dots'

Summer term of Year 2, if pupils are secure – bridge 100.

Year 3																																																			
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): +, add, addition, more, plus, make, sum, total, altogether, score, double, near double, one more, two more... ten more... one hundred more, how many more to make...? how many more is... than...? how much more is...?</p> <p>Instructional vocabulary: explain your method, explain how you got your answer, give an example of... show how you... show your working</p>																																																		
Objective and Strategies	Concrete	Pictorial	Abstract																																																
<p>Introduce column addition without crossing the boundary</p> <p>24 (20+4) +53 (50+3) 77 (70 + 7)</p> <p>Know the complements to 100. (For example 60 +40 = 100 AND 63 + 37 = 100).</p> <p>Introduce column addition with crossing the boundary</p>	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> 232 + 114 </div> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> </tr> <tr> <td style="text-align: center;">H</td> <td style="text-align: center;">T</td> <td style="text-align: center;">O</td> </tr> <tr> <td style="text-align: center;">  </td> <td style="text-align: center;">  </td> <td style="text-align: center;">  </td> </tr> </table> </div> <p>Make both numbers on a place value grid.</p> <div style="display: flex; align-items: center; margin-top: 20px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> </tr> <tr> <td style="text-align: center;">100</td> <td style="text-align: center;">10</td> <td style="text-align: center;">1</td> <td style="text-align: center;">146</td> </tr> <tr> <td style="text-align: center;">100</td> <td style="text-align: center;">10 10 10 10</td> <td style="text-align: center;">1 1 1 1 1 1 1 1</td> <td style="text-align: center;">+ 527</td> </tr> </table> </div> <p style="margin-top: 20px;">Add up the units and exchange 10 ones/units for one 10.</p> <div style="display: flex; align-items: center; margin-top: 20px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> </tr> <tr> <td style="text-align: center;">100</td> <td style="text-align: center;">10</td> <td style="text-align: center;">1</td> <td style="text-align: center;">146</td> </tr> <tr> <td style="text-align: center;">100</td> <td style="text-align: center;">10 10 10 10</td> <td style="text-align: center;">1 1 1 1 1 1 1 1</td> <td style="text-align: center;">+ 527</td> </tr> </table> </div>				H	T	O								100	10	1	146	100	10 10 10 10	1 1 1 1 1 1 1 1	+ 527					100	10	1	146	100	10 10 10 10	1 1 1 1 1 1 1 1	+ 527	<p>After practically using the base 10 blocks and place value counters, children can use bar models to represent the addition.</p> <div style="text-align: center; margin: 10px 0;">  </div> <div style="text-align: center; margin: 10px 0;"> <table border="1" style="border-collapse: collapse; width: 100px;"> <tr> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> <td style="width: 30px; height: 30px;"></td> </tr> <tr> <td style="text-align: center;">□</td> <td style="text-align: center;">+</td> <td style="text-align: center;">□ = □</td> </tr> <tr> <td style="text-align: center;">□</td> <td style="text-align: center;">+</td> <td style="text-align: center;">□ = □</td> </tr> </table> </div> <div style="text-align: center; margin: 10px 0;"> <table border="1" style="border-collapse: collapse; width: 150px;"> <tr> <td colspan="3" style="text-align: center; height: 30px;">?</td> </tr> <tr> <td style="width: 30px; text-align: center;">120</td> <td style="width: 30px; text-align: center;">132</td> <td style="width: 30px; text-align: center;">260</td> </tr> </table> </div>				□	+	□ = □	□	+	□ = □	?			120	132	260	<div style="text-align: center; margin: 10px 0;"> $\begin{array}{r} 625 \\ + 48 \\ \hline 13 \text{ (5+8)} \\ 60 \text{ (20 + 40)} \\ + 600 \text{ (600 + 0)} \\ \hline 673 \end{array}$ </div> <p>Expanded method first</p> <p>Then, use compact method without crossing a boundary, Then, compact with crossing a boundary to carry digits:</p> <div style="text-align: center; margin: 10px 0;"> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$ </div>
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St Bartholomew's MAT Calculation Policy



	<p>Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.</p> <p>This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.</p>		
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Year 4

Layers of vocabulary



Beck's Tiers of Vocabulary

Basic to subject specific (Beck's Tiers):

add, addition, more, plus, increase, sum, total, altogether score double, near double how many more to make...?

Instructional vocabulary:

calculate, work out, solve, investigate, question, answer, check

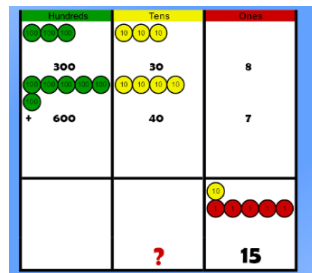
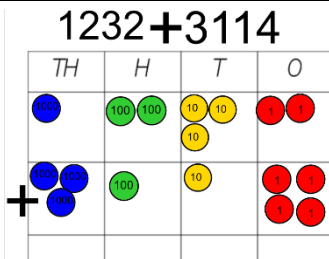
Objective and Strategies

Begin to use column addition without crossing the boundary using 4 digit numbers.

Begin to use column addition with crossing the boundary with 4 digit numbers.

Use column addition in the contexts of measures to include decimals.

Concrete



Crossing a boundary.



Use place value arrow cards to ensure that partitioning is understood.

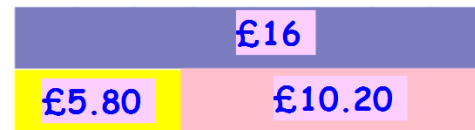
Pictorial

After practically using the base 10 blocks and place value counters, children can use bar models to represent the addition.

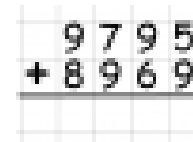


6509 + 2170 = _____

1890 - 362 = _____



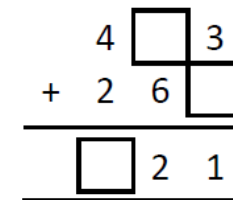
Abstract




Once confident with the method, analytical opportunities should be offered.

Calculation	Error	Correct solution
$\begin{array}{r} 1482 \\ + 672 \\ \hline 8202 \end{array}$		

Find the missing numbers in these calculations.



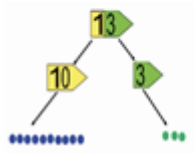








	Year 5	Year 6
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...?</p> <p>Instructional vocabulary: put, place arrange, rearrange change, change over split, separate</p>	<p>Basic to subject specific (Beck's Tiers): add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...?</p> <p>Instructional vocabulary: put, place arrange, rearrange change, change over adjusting, adjust split, separate, carry on, continue, repeat what comes next? predict describe the pattern, describe the rule, find, find all, find different investigate</p>

Subtraction

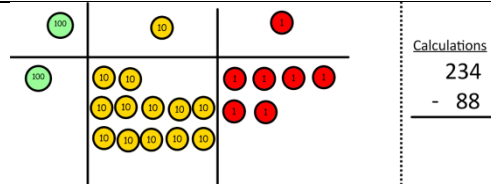
Year 1			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p> <p>Know that the number gets smaller because objects have been removed from the set.</p> <p>Practical models of subtraction.</p> <p>Concept of take away and counting back.</p>	<p>Basic to subject specific (Beck's Tiers): take away, distance between, difference between, less than. How many more? How much greater? How many fewer? how much more is...? – subtract, take (away), minus, leave, how many are left/left over? how many have gone? one less, two less, ten less... how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as</p> <p>Instructional vocabulary: start from, start with, start at, look at point, to show me</p> <p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p>$6 - 2 = 4$</p> <p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p> <p>$13 - 4$</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> <p>Use dienes to subtract larger numbers. E.g. $37 - 15$, $37 - 18$</p> <p>Compare amounts and objects to find the difference.</p> <p>Use cubes to build towers or make bars to find the difference</p>	<p>Cross out drawn objects to show what has been taken away.</p> <p>Count back on a number line or number track</p> <p>$15 - 3 = 12$</p> <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p> <p>Use basic bar models with items to find the difference</p>	<p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p> <p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p> <p>Tom has 5 pencils. Emma has pencils. How many more pencils does Tom have?</p>
<p>Concept of find the difference as counting on.</p>			

Year 2			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p> <p>Basic to subject specific (Beck's Tiers): subtract, subtraction, take (away), minus leave, how many are left/left over? one less, two less... ten less... one hundred less how many fewer is... than...? how much less is...? difference between half, half = equals, sign, is the same as, tens boundary difference, partition, rearrange, inverse, place value</p> <p>Instructional vocabulary: tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you...</p>	<p>Use a bead string to model counting back.</p> <p>What is the difference between 18 and 26?</p> <p>Link to addition.</p> $10 = 6 + 4$ $10 - 6 = 4$ <p>Use patterns to find answers to subtractions</p> $10 + 4 =$ $10 - 4 =$ $20 + 4 =$ $20 - 4 =$	<p>Comparison Bar Models</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p>	<p>$57 - 23 = 34$</p> <p>Partitioning the second number strategy</p> $76 - 43 =$ $76 - 40 = 36$ $36 - 3 = 33$ <p>When it is a subtraction calculation, underline the second number – this is the only number that can be partitioned.</p> $73 - 46 =$ $73 - 40 = 33$ $33 - 6 = 27$ <p><u>Find the difference</u> Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>

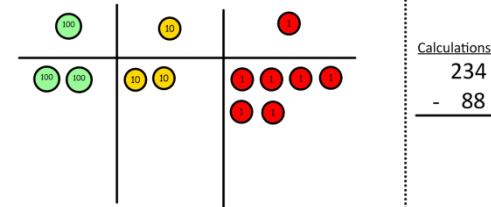
<p>2 digit subtract 2 digit.</p>	<p>Partitioning of numbers into T and U then HTU. Know what each digit represents.</p> <p>TU – TU HTU – TU</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>Build with dienes too.</p> </div>	<div style="border: 2px solid yellow; border-radius: 50%; padding: 10px; text-align: center; background-color: #fff9c4;"> <p>Think: I have 3 tens and 4 ones. I want to take away 9 ones.</p> </div> <div style="border: 2px solid blue; padding: 10px; margin-top: 10px;"> <p style="text-align: center; font-weight: bold; color: blue;">Workmat</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%; text-align: center;">Tens</th> <th style="width: 50%; text-align: center;">Ones</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">  </td> <td style="text-align: center;">  </td> </tr> </tbody> </table> </div>	Tens	Ones			
Tens	Ones						
							

Year 3			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): subtract, subtraction, take (away), minus, leave, how many are left/left over? one less, two less... ten less... one hundred less how many fewer is... than...? how much less is...? difference between half, half = equals, sign, is the same as, tens boundary, hundreds boundary, exchange, carried digits</p> <p>Instructional vocabulary: explain your method, explain how you got your answer, give an example of... show how you... show your working</p>		
<p>Column method without exchange.</p> <p>Column method with exchange.</p> <p>HTU – HTU</p> <p>The concept of zero as a place holder e.g. 406 has 6 units/ones and 40 tens which is the same as four hundred.</p>	<p>Use Base 10 to make the bigger number then take the smaller number away.</p> <p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p> <p>Make the larger number with the place value counters</p> <p>Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$ <p>Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.</p> <p>Now I can subtract my ones.</p> <p>Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred</p> <p>for ten tens.</p>	<p>Draw the</p> <p>Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ <p>dienes or place value counters alongside the written calculation to help to show working.</p> <p>Calculations</p> $\begin{array}{r} 176 - 64 = \\ 176 \\ - 64 \\ \hline 112 \end{array}$	<p>89 = 80 9 - 24 = 20 4 60 5 = 65</p> <p>653 - 321</p>

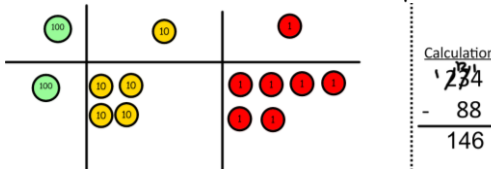
Now I can take away eight tens and complete my subtraction



$$\begin{array}{r} \text{Calculations} \\ 234 \\ - 88 \\ \hline \end{array}$$



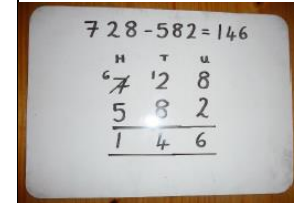
$$\begin{array}{r} \text{Calculations} \\ 234 \\ - 88 \\ \hline \end{array}$$



$$\begin{array}{r} \text{Calculations} \\ \cancel{2}34 \\ - 88 \\ \hline 146 \end{array}$$

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.


Children can start their formal written method by partitioning the number into clear place value columns.




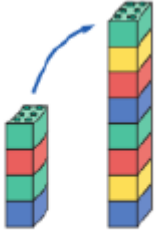
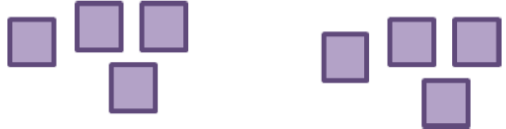
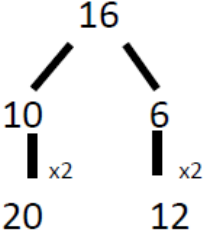
Moving forward the children use a more compact method.



Year 4							
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more/fewer is... than...? how much more/less is...? equals, sign, is the same as, tens boundary, hundreds boundary, inverse, exchange, carried digits</p> <p>Instructional vocabulary: calculate, work out, solve, investigate, question, answer, check</p>						
Objective and Strategies	Concrete	Pictorial	Abstract				
<p>Column method without exchange.</p> <p>Column method with exchange.</p> <p>4 digit subtract 4 digit.</p> <p>Apply method in the context of measures, including decimals.</p> <p>Continue the concept of zero as a place holder e.g. 5026 has 6 units/ones and 50 hundreds which is the same as five thousand.</p>	<p>Use place value cards to build to the numbers.</p> <p>Use cards to understand the decimal system.</p> <p>Use place counters for those who have not yet understand the exchange concept.</p>	<p>Bar modelling.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td colspan="2" style="text-align: center;">3682</td> </tr> <tr> <td style="width: 50%;">1245</td> <td style="width: 50%; text-align: center;">?</td> </tr> </table>	3682		1245	?	<p>Compact method:</p> $ \begin{array}{r} 6467 - 2684 \\ \begin{array}{r} 5131 \\ \cancel{6}467 \\ - 2684 \\ \hline 3783 \end{array} \end{array} $ $ \begin{array}{r} 3249 - 725 \\ \begin{array}{r} 1181 \\ 324\cancel{9}0 \\ - 725 \\ \hline 31765 \end{array} \end{array} $ <p>Look at the accurate exchanging with more than one zero:</p> $ \begin{array}{r} 6003 \\ - 2786 \\ \hline \end{array} $
3682							
1245	?						

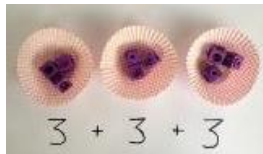
	Year 5	Year 6
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): subtract, subtraction, take (away), minus, leave, how many are left/left over? ten less... one hundred less how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as tens boundary, hundreds boundary, inverse, units boundary, tenths boundary, exchange, carried digits</p> <p>Instructional vocabulary: put, place, arrange, rearrange change, change over, adjusting, adjust, split, separate</p>	<p>Basic to subject specific (Beck's Tiers): subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? difference between half, halve how many more/fewer is... than...? how much more/less is...? equals, sign, is the same as tens boundary, hundreds boundary, units boundary, tenths boundary, inverse</p> <p>Instructional vocabulary: put, place arrange, rearrange change, change over adjusting, adjust split, separate, carry on, continue, repeat, what comes next? Predict, describe the pattern, describe the rule, find, find all, find different, investigate</p>

Multiplication

Year 1			
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): count in ones, twos... tens... array, groups of, equal groups, odd, even</p> <p>Instructional vocabulary: carry on, continue repeat what comes next? find, choose, collect, use, make, build tell me, describe, pick out, talk about, explain, show me, read, write, record</p>		
Objective and Strategies	Concrete	Pictorial	Abstract
<p>To understand the concept of doubling.</p> <p>Counting in steps of 2s, 5s, 10s.</p> <p>To understand that multiplication is repeated addition.</p>	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>

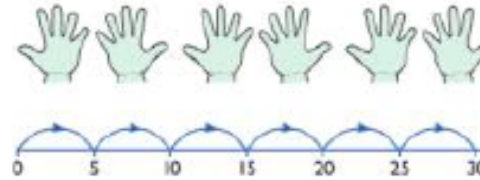


Count in multiples supported by concrete objects in equal groups.



Use different objects to add equal groups.

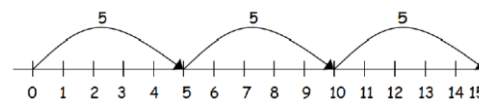
Use a number line or pictures to continue support in counting in multiples.



There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



2 add 2 add 2 equals 6



$5 + 5 + 5 = 15$

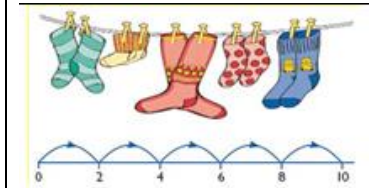
Count in multiples of a number aloud.

Write sequences with multiples of numbers.

2, 4, 6, 8, 10

5, 10, 15, 20, 25, 30

Write addition sentences to describe objects and pictures.



$2 + 2 + 2 + 2 + 2 = 10$

$2 \times 5 = 10$



2 multiplied by 5

5 pairs


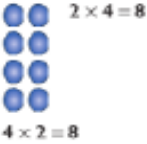
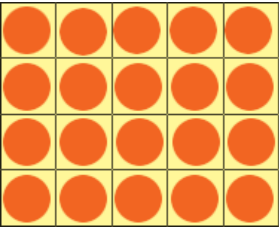
5 hops of 2

To use arrays to show that multiplication is commutative.

Create arrays using counters/ cubes to show multiplication sentences.





Draw arrays in different rotations to find **commutative** multiplication sentences.



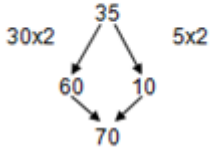



































Link arrays to area of rectangles.


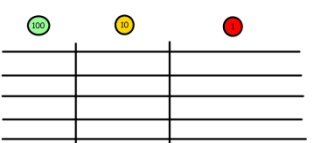
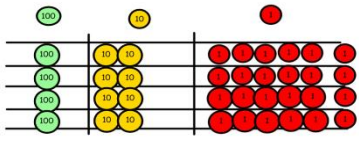
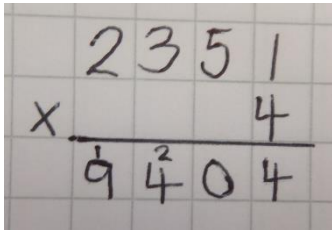
Use an array to write multiplication sentences and reinforce repeated addition.





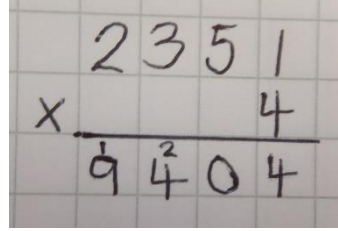


$5 + 5 + 5 = 15$
 $3 + 3 + 3 + 3 + 3 = 15$
 $5 \times 3 = 15$
 $3 \times 5 = 15$

Year 2									
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): lots of, groups of \times, times, multiply, multiplied by multiple of once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally</p> <p>Instructional vocabulary: carry on, continue, repeat, what comes next? predict describe the pattern describe the rule find, find all, find different, investigate</p>								
	Objective and Strategies	Concrete	Pictorial	Abstract					
<p>Know tables facts for 2s, 10s and 5s and begin 3s and 4s.</p> <p>To be able to partition a 2 digit number. E.g. 12×5 is...</p> <p>10×5 Add 2×5</p> <p>Doubles are the same as multiplying by 2.</p>	<p>Use counters, pegs boards, money to build arrays.</p>	<p>Consolidate arrays and repeated addition. Recalling facts. $4 \times 5 = 20, 5 \times 4 = 20.$</p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td colspan="3" style="text-align: center;">12</td> </tr> <tr> <td style="text-align: center;">4</td> <td style="text-align: center;">4</td> <td style="text-align: center;">4</td> </tr> </table>	12			4	4	4	<p>Partitioning strategy for doubling.</p> <p>Double 35</p>  <p>Know that 3×4 is that same as $4 + 4 + 4$</p>
12									
4	4	4							

Year 3																														
Objective and Strategies	Concrete	Pictorial	Abstract																											
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p> <p>Know times tables for: 2, 3, 4, 5, 8, 10.</p> <p>Understand multiplying by 10.</p> <p>Understand that multiplying a number by zero, the answer will always be zero.</p>	<p>Basic to subject specific (Beck's Tiers): lots of, groups of \times, times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each...</p> <p>Instructional vocabulary: carry on, continue, repeat what comes next? Predict, describe the pattern, describe the rule, find, find all, find different, investigate, choose, decide, collect</p> <p>Show the link with arrays to introduce partitioning for multiplication.</p> <table border="1" data-bbox="436 657 795 790"> <tr> <td>x</td> <td>10</td> <td>3</td> <td></td> </tr> <tr> <td>4</td> <td colspan="2">  </td> <td>4 rows of 10</td> </tr> <tr> <td></td> <td colspan="2">  </td> <td>4 rows of 3</td> </tr> </table> <p>Move on to using Base 10/dienes to move towards a more compact method.</p> <p>4 rows of 13</p> <table border="1" data-bbox="436 957 728 1109"> <tr> <td>x</td> <td>T</td> <td>U</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table>	x	10	3		4			4 rows of 10				4 rows of 3	x	T	U													<p>Children can represent the work they have done with place value counters in a way that they understand.</p> <p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking.</p>	<p>Start with multiplying two digit by one digit numbers</p> <p>"20 multiplied by 3 equals 60 and 7 multiplied by 3 equals 21. 60 add 21 equals 81."</p> <p>23 \times 8 = 20 \times 8 = 160 3 \times 8 = 24</p> <p>.</p> <p>23 \times 8 24 (8 \times 3) 180 (8 \times 20) <u>184</u></p>
x	10	3																												
4			4 rows of 10																											
			4 rows of 3																											
x	T	U																												
																														
																														
																														
																														

Year 4			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p> <p>To know all multiplication facts up to 12 x 12.</p> <p>To know how to multiply by 10 and 100.</p> <p>To understand distributive law.</p> <p>Begin to use short multiplication method (short is when there is a single multiplier).</p>	<p>Basic to subject specific (Beck's Tiers): lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve, factor, multiple</p> <p>Instructional vocabulary: carry on, continue, repeat what comes next? predict, describe the pattern, describe the rule pattern, puzzle, calculate, calculation, mental calculation, method, jotting, answer right, correct, wrong, what could we try next? how did you work it out? number sentence, sign, operation, symbol, equation</p> <p>Fill each row with 126. Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</p>  <p>Calculations 4 x 126</p> <p>Add up each column, starting with the ones making any exchanges needed.</p>  <p>Calculations 4 x 126</p>	<p>Children can represent the work they have done with place value counters in a way that they understand.</p> <p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking.</p>	<p>Start with expanded short multiplication, reminding the children about lining up their numbers clearly in columns. See below:</p> $\begin{array}{r} 346 \\ \times 9 \\ \hline 54 \text{ (} 9 \times 6 \text{)} \\ 360 \text{ (} 9 \times 40 \text{)} \\ 2700 \text{ (} 9 \times 300 \text{)} \\ \hline 3114 \end{array}$ <p>Once confident, move to compact notation:</p> 

Year 5																					
Objective and Strategies	Concrete	Pictorial	Abstract																		
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p> <p>To know all multiplication facts up to 12 x 12.</p> <p>To know how to multiply by 10, 100 and 1000.</p> <p>Begin to use long multiplication method (short is when there is a single multiplier).</p> <p>Move to working with decimals.</p> <p>multiply numbers with up to two decimal places by whole numbers (year 6)</p>	<p>Basic to subject specific (Beck's Tiers): lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally factor, multiple, prime, composite</p> <p>Instructional vocabulary: carry on, continue, repeat what comes next? predict, describe the pattern, describe the rule find, find all, find different, investigate</p> <p>Understanding the effect of multiplying by 10, 100 and 1000. Create a visual place value chart and model numbers physically moving when multiplying/dividing by multiples of 10.</p> 	<p>Moving forward, multiply by a 2 digit number showing the different rows within the calculation</p> <p>Use a laminated place value resource to assist children in multiplying/dividing by multiples of 10.</p> <p style="text-align: center;">Multiplying and Dividing by 10, 100 and 1000</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>10 000</td> <td>1000</td> <td>100</td> <td>10</td> <td>1</td> <td>●</td> <td>$\frac{1}{10}$</td> <td>$\frac{1}{100}$</td> <td>$\frac{1}{1000}$</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td>●</td> <td></td> <td></td> <td></td> </tr> </table> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p>Multiplying</p> <p>X 10 digits move LEFT 1 space X 100 digits move LEFT 2 spaces X 1000 digits move LEFT 3 spaces</p>  </div> <div style="text-align: center;"> <p>Dividing</p> <p>÷ 10 digits move RIGHT 1 space ÷ 100 digits move RIGHT 2 spaces ÷ 1000 digits move RIGHT 3 spaces</p>  </div> </div>	10 000	1000	100	10	1	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$						●				<p>Compact notation: (by 1 digit)</p>  <p>Expanded long method:</p> <div style="border: 1px solid black; padding: 5px; margin-top: 5px;"> <p>Expanded</p> <p>78 x 42 ----- 16 (2 x 8) 140 (2 x 70) 320 (40 x 8) +2800 (40 x 70) ----- 3276</p> </div>
10 000	1000	100	10	1	●	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$													
					●																



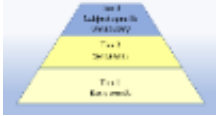
$$\begin{array}{r}
 2336 \\
 \times \quad 54 \\
 \hline
 9344 \\
 116800 \leftarrow \text{Place holder} \\
 \hline
 126144 \\
 11
 \end{array}$$

Expanded long with decimals



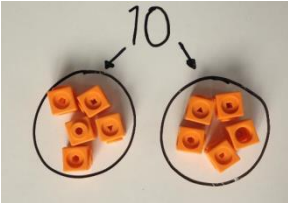
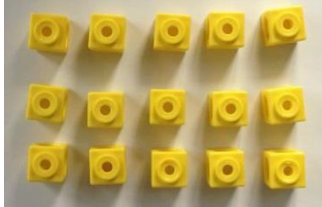
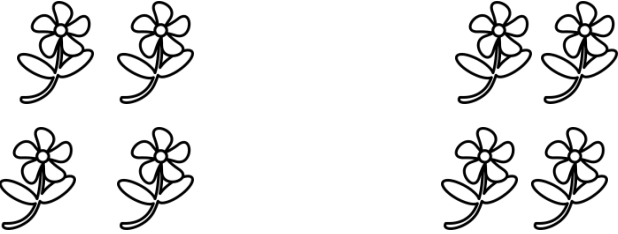

$$\begin{array}{r}
 3.45 \\
 \times \quad 6 \\
 \hline
 0.30 \\
 2.40 \\
 18.00 \\
 \hline
 20.70
 \end{array}$$

Compact with decimals

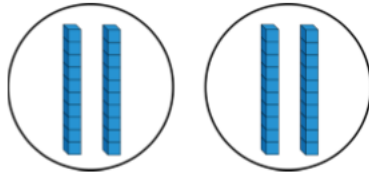
$$\begin{array}{r}
 3.59 \\
 \times 7 \\
 \hline
 25.13 \\
 46
 \end{array}$$

	Year 6
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally factor, multiple, prime, composite</p> <p>Instructional vocabulary: carry on, continue, repeat what comes next? predict, describe the pattern, describe the rule find, find all, find different, investigate</p>

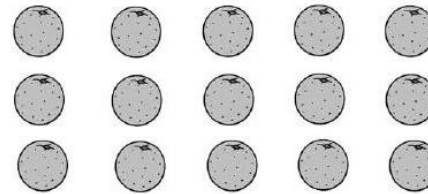
Division

Year 1			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): count in ones, twos... tens... share, groups of, equal groups odd, even</p> <p>Instructional vocabulary: count out, share out, left, left over</p>		
<p>To understand that division is sharing into equal groups.</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>   <p>I have got 3 groups of 5 or 5 groups of 3.</p>	<p>Children use pictures or shapes to share quantities.</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $8 \div 2 = 4$ </div> <p>Children represent objects by drawing circles and dots.</p> 	<p>Share 9 buns between three people.</p> $9 \div 3 = 3$

Year 2			
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of +, divide, divided by, divided into left, left over</p> <p>Instructional vocabulary: tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you</p>		
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Division as grouping and sharing.</p> <p>To know that division non commutative (they should know to put the biggest number first).</p> <p>Know that halving is the same as divide by 2.</p> <p>Concept of the 'leftover' leading to understanding of the remainder.</p> <p>The remainder to be dealt with depending on the context (i.e. what could you do with the remainder? Cake you could split. A pencil you couldn't).</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p> <p>There are 7 cakes and 2 children. How many cakes will they each get? 'Leftovers' introduced.</p>	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p> <p>$20 \div 5 = ?$ $5 \times ? = 20$</p>	<p>$20 \div 5 = 4$</p> <p>Divide 25 into 5 groups. How many are in each group?</p> <p>Find the inverse of multiplication and division</p>




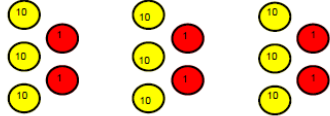

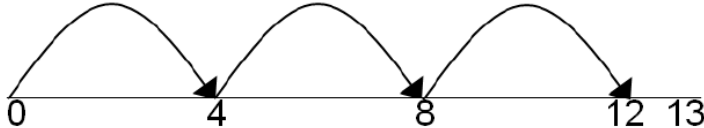
$$40 \div 2 = 20$$




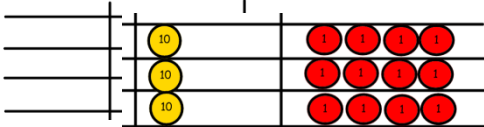
15 divided by
 $3 = 5$
15 divided by
 $5 = 3$

sentences by creating four linking number sentences.

$$7 \times 5 = 35$$
$$5 \times 7 = 35$$
$$35 \div 7 = 5$$
$$35 \div 5 = 7$$

Year 3																			
Objective and Strategies	Concrete	Pictorial	Abstract																
<p>Layers of vocabulary</p>  <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of ÷, divide, division, divided by, divided into left, left over, remainder, dividend, divisor</p> <p>Instructional vocabulary: calculate, work out, solve, investigate, question, answer, check</p>																		
<p>To understand division as sharing and grouping.</p> <p>To know when a remainder will occur, and how to write it using 'r' notation.</p> <p>To know how to rearrange the dividend in the multiples of the divisor.</p>	<p>$96 \div 3 = 32$</p>  <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Use place value counters to build the dividend (in this example this is 96).</p> </div>	<p>$63 \div 3$</p>  <p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p> 	<div style="border: 1px solid black; padding: 5px;"> <p>Use partitioning/re-arranging to find multiples of the divisor.</p> <p>$48 \div 3 =$</p> <p>'What do I know about 3 x tables?'</p> <p>"I know $3 \times 10 = 30$."</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">30</td> <td style="text-align: center;">18</td> </tr> <tr> <td style="text-align: center;">↓</td> <td style="text-align: center;">↓</td> </tr> <tr> <td style="text-align: center;">10</td> <td style="text-align: center;">6</td> </tr> </table> <p>$48 \div 3 = 16$</p> <p>$10 \times 3 = 30$ $6 \times 3 = 18$</p> </div> <p>Complete written divisions and show the remainder using r.</p> <p style="text-align: center;">$29 \div 8 = 3 \text{ REMAINDER } 5$</p> <table style="margin-left: auto; margin-right: auto; text-align: center;"> <tr> <td>↑</td> <td>↑</td> <td>↑</td> <td></td> <td>↑</td> </tr> <tr> <td>dividend</td> <td>divisor</td> <td>quotient</td> <td></td> <td>remainder</td> </tr> </table>	30	18	↓	↓	10	6	↑	↑	↑		↑	dividend	divisor	quotient		remainder
30	18																		
↓	↓																		
10	6																		
↑	↑	↑		↑															
dividend	divisor	quotient		remainder															

Year 4			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p> <p>To be able to use short division (this is with a single digit divisor).</p> <p>Continue to use the rearranging the dividend method.</p>	<p>Basic to subject specific (Beck's Tiers): share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of ÷, divide, division, divided by, divided into left, left over, remainder, dividend, divisor</p> <p>Instructional vocabulary: calculate, work out, solve, investigate question, answer, check</p> <p>Tens Units</p> <p>3 2</p> <p>3</p> <p>$96 \div 3 = 32$</p> <p>Use place value counters to divide using the bus stop method alongside</p> <p>Calculations $42 \div 3$</p> <p>$42 \div 3 =$ Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>	<p>Continue to develop partitioning/re-arranging to find multiples of the divisor.</p> <p>$96 \div 6$ "What do I know? $6 \times 10 = 60$"</p> <p>$96 \div 6 = 16$</p> <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no carrying.</p> <p>Then move to divisions with carrying which do not result in a remainder.</p> <p>Move onto divisions with a remainder.</p>

	<p style="text-align: center;">  </p> <p style="text-align: right;">We</p>  <p>exchange this ten for ten ones and then share the ones equally among the groups.</p> <p>We look how much in 1 group so the answer is 14.</p>		
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Year 5			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p>	<p>Basic to subject specific (Beck's Tiers): equal groups of, divide, division, divided by, divided into remainder, factor, quotient, divisible by, inverse</p> <p>Instructional vocabulary: calculate, work out, solve, investigate question, answer, check same, different missing number/s number facts, number pairs, number bonds, greatest value, least value</p>		
<p>To be able to use short division (this is with a single digit divisor), with up to 4 digit dividends.</p> <p>To apply my knowledge of the tests of divisibility.</p> <p>To divide by 10, 100 and 1000 mentally.</p> <p>Continue to use the rearranging the dividend method.</p>	<p>Understanding the effect of dividing by 10, 100 and 1000.</p> <p>Go back and use place value counters if children do not understand (see year 3 division).</p>	$847 \div 7$ <p>"What do I know? I know $7 \times 12 = 84$ so $7 \times 120 = 840$"</p> $\begin{array}{r} 847 \\ 840 \quad 7 \\ \downarrow \quad \downarrow \\ 120 \quad 1 \end{array}$ $847 \div 7 = 121$ <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>To complete divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \\ \underline{54} \\ 32 \\ \underline{30} \\ 2 \end{array}$ $\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 53509} \end{array}$

Year 6			
Objective and Strategies	Concrete	Pictorial	Abstract
<p>Layers of vocabulary</p> <p>Beck's Tiers of Vocabulary</p> <p>To be able to use short (this is with a single digit divisor), and long division (2 digit divisor) with up to 4 digit dividends and decimals.</p> <p>To apply my knowledge of the tests of divisibility.</p> <p>To divide by 10, 100 and 1000 mentally.</p> <p>Continue to use the rearranging the dividend method.</p>	<p>Basic to subject specific (Beck's Tiers): equal groups of, divide, division, divided by, divided into remainder, factor, quotient, divisible by, inverse, remainders as fractions or decimals</p> <p>Instructional vocabulary: calculate, work out, solve, investigate question, answer, check, same, different missing number/s number facts, number pairs, number bonds greatest value, least value</p> <p>$17 \div 5 = 3 \frac{2}{5}$</p> <p>Use the number line to explore remainders and expressing the quotient as a fraction or decimal.</p>	<p>$581 \div 7 =$</p> $\begin{array}{r} 560 + 21 = 581 \\ \downarrow \quad \downarrow \\ 80 + 3 = 83 \end{array}$ <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>long division $560 \div 24$</p> $\begin{array}{r} 23 \text{ r } 8 \\ 24 \overline{) 560} \\ \underline{-48} \\ 80 \\ \underline{-72} \\ 8 \end{array}$ <p>remainder as a whole number</p> $\begin{array}{r} 23 \frac{8}{24} (1/3) \\ 24 \overline{) 560} \\ \underline{-48} \\ 80 \\ \underline{-72} \\ 8 \end{array}$ <p>remainder as a fraction in its lowest form</p> $\begin{array}{r} 23.3 \\ 24 \overline{) 560.0} \\ \underline{-48} \\ 80 \\ \underline{-72} \\ 80 \\ \underline{-72} \\ 8 \end{array}$ <p>remainder as a decimal</p>